Student Workbook

Unit Review — Systems, Matrices & Determinants

Name: ________________________________ Date: _________________ Class: _________

UNIT SUMMARY

Consolidate your understanding of two- and three-variable linear systems, nonlinear systems, partial fractions, matrix operations, Gaussian elimination, solving with inverses, and Cramer's Rule.

Unit Summary: Systems of Equations and Matrices

$$Ax = b \mid \det(A) = ad - bc \mid x = D_x/D$$

9.1	Two-Variable Systems — substitution, elimination, graphing. Unique / no solution / infinitely many.
9.2	Three-Variable Systems — elimination to triangular form, back-substitution.
9.3	Nonlinear Systems — substitution for curves; graph inequality regions (test a point).
9.4	Partial Fractions — 4 cases (distinct linear, repeated linear, irreducible quadratic, repeated quadratic). Improper → long division first.
9.5	Matrix Operations — add/subtract (same size), scalar mult, matrix mult (row × col). AB ≠ BA.
9.6	Gaussian Elimination — augmented matrix [A|b], row operations, REF → back-sub, RREF → read directly.
9.7	Inverse Method — $\det(A)=ad-bc$; $A^{-1}=(1/\det)\cdot[[d,-b],[-c,a]]$; $x=A^{-1}b$. Gauss-Jordan for larger.
9.8	Cramer's Rule — $x=D_x/D$, $y=D_y/D$. Replace column with constants. D=0 → no unique solution.

Review Examples

EXAMPLE 1

Solve the system: 3x + 2y = 12, x − y = 1

- From eq2: $x = y + 1$
- Substitute: $3(y+1) + 2y = 12 \rightarrow 5y = 9 \rightarrow y = 9/5$
- $x = 9/5 + 1 = 14/5$

Answer: (x, y) = (14/5, 9/5)

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EXAMPLE 2

Decompose: (2x + 3) / ((x+1)(x−1))

- Set up: A/(x+1) + B/(x−1). Multiply: 2x+3 = A(x−1) + B(x+1)
- x = 1: 5 = 2B → B = 5/2
- x = −1: 1 = −2A → A = −1/2

Answer: (−1/2)/(x+1) + (5/2)/(x−1)

EXAMPLE 3

Find AB where A = [[1,2],[3,4]] and B = [[2,0],[1,3]]

- c_{11} = 1·2 + 2·1 = 4; c_{12} = 1·0 + 2·3 = 6
- c_{21} = 3·2 + 4·1 = 10; c_{22} = 3·0 + 4·3 = 12

Answer: AB = [[4,6],[10,12]]

EXAMPLE 4

Solve using Gaussian elimination: x + 2y + z = 8, 2x + y − z = 3, x − y + 2z = 5

- Augmented: [[1,2,1|8],[2,1,−1|3],[1,−1,2|5]]
- $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$: [[1,2,1|8],[0,−3,−3|−13],[0,−3,1|−3]]
- $R_3 \to R_3 - R_2$: [[1,2,1|8],[0,−3,−3|−13],[0,0,4|10]] → z=5/2, y=11/6, x=11/6

Answer: x = 11/6, y = 11/6, z = 5/2

EXAMPLE 5

Solve 4x + y = 9, 3x + 2y = 8 using Cramer's Rule

- D = det([[4,1],[3,2]]) = 8 − 3 = 5
- D_x = det([[9,1],[8,2]]) = 18 − 8 = 10 → x = 10/5 = 2
- D_y = det([[4,9],[3,8]]) = 32 − 27 = 5 → y = 5/5 = 1

Answer: x = 2, y = 1

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Solve: 2x − y = 5, x + 3y = 7.

Hint: Use substitution or elimination. From eq1: y = 2x − 5.

2 Decompose: (x + 5) / (x(x + 5)).

Hint: Simplify first: (x+5)/(x(x+5)) = 1/x. No decomposition needed!

3 Find A^{-1} for A = [[5,3],[2,1]].

Hint: det = 5·1 − 3·2 = −1; A^{-1} = (1/−1)·[[1,−3],[−2,5]].

4 Write the augmented matrix for: 2x + y − z = 4, x − 2y + 3z = 1, 3x + y + z = 7. Then perform two row operations.

Hint: Swap R_1 and R_2 first to get a leading 1.

5 Evaluate: det([[2,3,1],[0,4,−1],[1,2,3]]).

Hint: Expand along column 1 (it has a zero): 2·det([[4,−1],[2,3]]) − 0 + 1·det([[3,1],[4,−1]]).

Quick Review Quiz

Circle the letter of the correct answer for each question.

1 **Which method is most efficient when solving Ax=b for 5 different b vectors with the same A?** Multiple Choice

- **A** Substitution
- **B** Gaussian elimination each time
- **C** Inverse method (compute A^{-1} once)
- **D** Cramer's Rule

2 **The partial fraction form for $1/((x-1)^2(x+2))$ is:** Multiple Choice

- **A** $A/(x-1)^2 + B/(x+2)$
- **B** $A/(x-1) + B/(x-1)^2 + C/(x+2)$
- **C** $(Ax+B)/(x-1)^2 + C/(x+2)$
- **D** $A/(x-1) + B/(x+2)$

3 **A row [0 0 0 | 5] in a reduced augmented matrix means:** Multiple Choice

- **A** z = 5
- **B** Free variable
- **C** No solution
- **D** Infinitely many solutions

4 **For A = [[3,1],[7,2]], det(A) =** Multiple Choice

- **A** −1
- **B** 1
- **C** 6
- **D** 13

5 **Matrix multiplication AB is defined when:** Multiple Choice

A A and B have the same dimensions

B The number of rows of A equals the number of columns of B

C The number of columns of A equals the number of rows of B

D A and B are both square

Common Mistakes to Avoid

✗ Forgetting to check det(A) before using the inverse method or Cramer's Rule.

✓ Always compute det(A) first. If det=0, neither method works — use Gaussian elimination instead.

✗ Using a constant numerator for an irreducible quadratic factor in partial fractions.

✓ Irreducible quadratic factors (x^2+bx+c) require a LINEAR numerator Ax+B, not just a constant A.

✗ Multiplying matrices entry-by-entry like addition.

✓ Matrix multiplication uses the row-times-column dot product. Entry c_{ij} = (row i of A)·(column j of B).

✗ Applying a row operation to only part of the row (forgetting the augmented column).

✓ Row operations apply to the ENTIRE row, including the constant column in the augmented matrix.

✗ Assuming AB = BA for matrices.

✓ Matrix multiplication is NOT commutative. AB and BA are generally different matrices.

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Math Tips

★ Choose your method wisely: substitution/elimination for 2×2, Gaussian for 3×3+, inverse for repeated b vectors, Cramer's for one variable at a time.

★ For partial fractions: factor the denominator completely first, then set up the correct form based on the factor types.

★ Dimension check for matrix multiplication: $(m\times n)(n\times p) = m\times p$. The inner dimensions must match.

★ In Gaussian elimination, work column by column left to right. Get a leading 1, then eliminate all entries below it.

★ The determinant is the key to everything: $\det \neq 0 \rightarrow$ unique solution, invertible matrix, Cramer's Rule works.

★ For nonlinear systems, substitution is almost always the right approach — solve one equation for one variable and substitute.